

On the occasion of the 110th anniversary since the birth of
N. N. Bogolyubov.

CONFERENCE ON NONLINEARITY

Belgrade 11—12. 10. 2019

Book of Abstracts

Organizer:

Serbian Academy of Nonlinear Sciences (SANS)

Coorganizers:

Mathematical Institute of SASA, and
Southeastern European Network in Mathematical and
Theoretical Physics (SEENET-MTP)

More information on <http://imqf.ekof.bg.ac.rs/ConfNonlin/>.

ISBN: 978-86-905633-6-4

Title: Conference on Nonlinearity, Book of Abstracts

Editors: Branko Dragovich and Željko Čupić

Published by: Serbian Academy of Nonlinear Sciences, Kneza Mihaila 36, Beograd

Printed by: „Jovan“ – Printing and publishing company, 40 Copies

Number of pages: 32, Format A4, Printing finished in October 2019

PROGRAM COMMITTEE

- Milivoj Belić (Texas A & M University in Qatar, Doha)
- Milan Dimitrijević (Belgrade Observatory and SANS, Serbia)
- Branko Dragovich (President of SANS)
- Vladan Đorđević (SASA and SANS)
- Branislav Sazdović (Institute of Physics, University of Belgrade, and SANS)
- Branko Urošević (Faculty of Economics, University of Belgrade, and SANS)

ORGANIZING COMMITTEE

- Branko Dragovich, Chairman
- Željko Čupić, IHTM, Vice Chairman
- Ćemal Dolićanin (State University in Novi Pazar)
- Goran Djordjevic (Faculty of Mathematics and Natural Sciences, University of Niš)
- Božidar Jovanović (MI SASA, Belgrade)
- Nikolaj Ostrovski (HIPOL, Serbia)

SPEAKERS

1. Ivana Atanasovska (Belgrade, Serbia)
2. Milivoj Belić (Doha, Qatar)
3. Zeljko Cupic (Belgrade, Serbia)
4. Ivan Dimitrijevic (Belgrade, Serbia)
5. Goran Djordjevic (Niš, Serbia)
6. Branko Dragovich (Belgrade, Serbia)
7. Nenad Filipović (Kragujevac, Serbia)
8. Katica (Stevanović) Hedrih (Niš, Serbia)
9. Deszo Horvath (Szeged, Hungary)
10. Božidar Jovanović (Belgrade, Serbia)
11. Ljiljana Kolar-Anić (Belgrade, Serbia)
12. Wieslaw Krolikowsky (Doha, Qatar)
13. Stevan Maćešić (Szeged, Hungary)
14. Aleksandar Obradović (Belgrade, Serbia)
15. Nikolaj Ostrovski (Odžaci, Serbia)
16. Papp Paszkal (Szeged, Hungary)
17. Miguel Sanjuan (Madrid, Spain)
18. Branislav Sazdovic (Belgrade, Serbia)
19. Vladimir Skarka (Angers, France)
20. Marko Stojanović (Niš, Serbia)
21. Branko Urošević (Belgrade, Serbia)
22. Anatoly Zagorodny (Kiev, Ukraine)
23. Volodymyr Zasenka (Kiev, Ukraine)
24. Slobodan Zdravković (Belgrade, Serbia)

PRELIMINARY LIST OF PARTICIPANTS

1. Slobodan Anic (Belgrade, Serbia)
2. Ivana Atanasovska (Belgrade, Serbia)
3. Milivoj Belić (Doha, Qatar)
4. Zeljko Cupic (Belgrade, Serbia)
5. Ivan Dimitrijevic (Belgrade, Serbia)
6. Milan Dimitrijević (Belgrade Serbia)
7. Goran Djordjevic (Niš, Serbia)
8. Vladan Djordjevic (Belgrade, Swerbia)
9. Branko Dragovich (Belgrade, Serbia)
10. Nenad Filipović (Kragujevac, Serbia)
11. Katica (Stevanović) Hedrih (Niš, Serbia)
12. Deszo Horvath (Szeged, Hungary)
13. Božidar Jovanović (Belgrade, Serbia)
14. Ljiljana Kolar-Anić (Belgrade, Serbia)
15. Wieslaw Krolikowsky (Doha, Qatar)
16. Stevan Maćešić (Szeged, Hungary)
17. Vladimir Markovic (Belgrade, Serbia)
18. Nataša Mišić (Belgrade, Serbia)
19. Aleksandar Obradović (Belgrade, Serbia)
20. Nikolaj Ostrovski (Odžaci, Serbia)
21. Papp Paszkal (Szeged, Hungary)
22. Miguel Sanjuan (Madrid, Spain)
23. Branislav Sazdovic (Belgrade, Serbia)
24. Vladimir Skarka (Angers, France)
25. Ana Stanojevic (Belgrade, Serbia)
26. Marko Stojanović (Niš, Serbia)
27. Branko Urošević (Belgrade, Serbia)
28. Anatoly Zagorodny (Kiev, Ukraine)
29. Volodymyr Zasenka (Kiev, Ukraine)
30. Slobodan Zdravković (Belgrade, Serbia)

THE INTRODUCTION OF MULTI-BODY CONTACT IN NON-LINEAR DYNAMICS

Ivana Atanasovska, Mathematical Institute of SASA, Belgrade, Serbia

Email: iatanasovska@mi.sanu.ac.rs

The definition of multi-body contact as new approach will be given, with accent on the applications in non-linear dynamics of complex mechanical systems. The multi-body contact is new original term defined by the author and presents the phenomenon of specific behavior of few deformable bodies mutually in contact when they form a system which is a part of other more complex mechanical system loaded with complex loads and/or excitations and subjected to non-linear dynamics. The new idea presented in this paper is inspired by the real mechanical systems completely different at first glance, but with the same multi-body contact characteristics as a part of a large mechanical system subjected to dynamics behavior. The total complex stiffness and total complex damping were recognized for each of these systems as main influence characteristics on dynamics of complex systems which part they are. The new definition and theoretical consideration presented in this paper could be a new step in unification of the non-linear dynamics analysis for real complex mechanical systems with multi-body contact parts. Except the theoretical definitions of this new term, in the paper few real systems as examples will be shown to illustrate the introduced approach.

Rogue waves in optics: Facts and fictions

Milivoj R Belić,¹ Stanko N Nikolić,²

¹ Texas A&M University at Qatar, 23874 Doha, Qatar

² Institute of Physics Belgrade, Pregrevica 118, 10080 Zemun, Serbia

Email: milivoj.belic@qatar.tamu.edu

Rogue waves are giant nonlinear waves that suddenly appear and disappear in oceans and optics. We discuss the facts and fictions related to their strange nature, dynamic generation, ingrained instability, and potential applications. We propose the method of mode pruning for suppressing the modulation instability of rogue waves. We demonstrate how to produce Talbot carpets – ordered recurrent images of light and plasma waves – by rogue waves, for possible use in nanolithography.

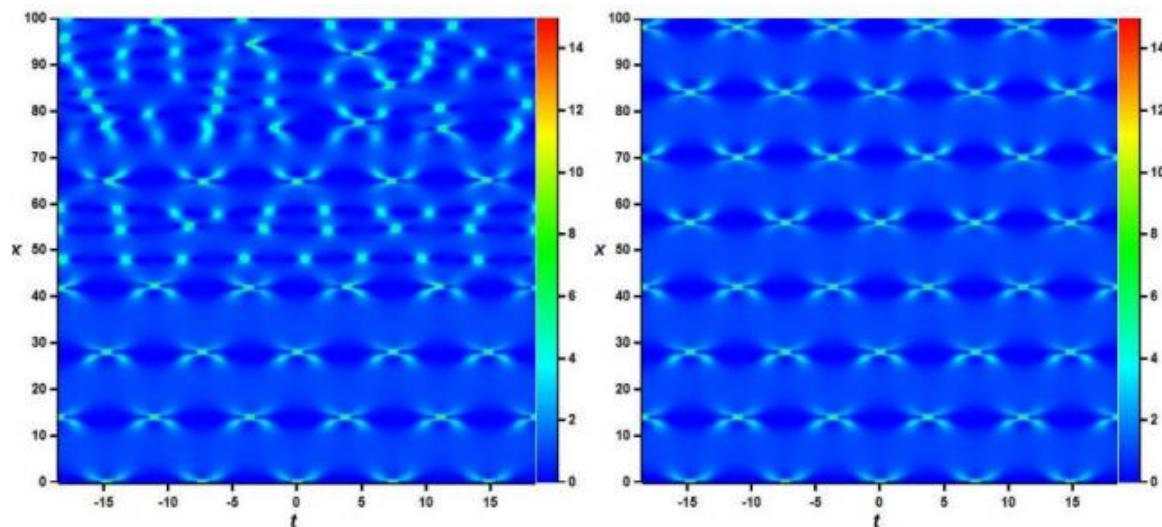


Figure 1: Unstable and stabilized Talbot carpets, made from secondorder rogue waves produced in the nonlinear Schrödinger equation.

References

1. N Akhmediev, B Kibler, F Baronio, M Belić, *et al.*, "Roadmap on optical rogue waves and extreme events." *J. Optics*, 18 (6), 063001 (2016).
2. SA Chin, OA Ashour, MR Belić, "Anatomy of the Akhmediev breather: Cascading instability, first formation time, and Fermi-Pasta-Ulam recurrence." *PHYSICAL REVIEW E* 92, 0632025 (2015).
3. SA Chin, OA Ashour, SN Nikolić, MR Belić, "Maximal intensity higherorder Akhmediev breathers of the nonlinear Schrödinger equation and their systematic generation." *Phys. Letters A* 380, 3625 (2016).
4. SA Chin, OA Ashour, SN Nikolić, MR Belić, "Peak-height formula for higher-order breathers of the nonlinear Schrodinger equation." on nonuniform backgrounds." *PHYSICAL REVIEW E* 95, 012211 (2017).
5. SN Nikolić, OA Ashour, NB Aleksić, MR Belić, SA Chin, "Breathers, solitons and rogue waves of the quintic nonlinear Schrödinger equation on various backgrounds." *Nonlinear Dyn* <https://doi.org/10.1007/s11071-018-4726-8> (2019).

Instability region in models of nonlinear reaction systems. The Stoichiometric Network Analysis

Željko Čupić,¹ Stevan Maćešić^{2,3} and Ljiljana Kolar-Anić²

¹ *University of Belgrade, Institute of Chemistry, Technology and Metallurgy, Department of Catalysis and Chemical Engineering, Belgrade, Serbia;*

² *University of Belgrade, Faculty of Physical Chemistry, Belgrade, Serbia;*

³ *University of Szeged, Department of Physical Chemistry and Materials Science, Szeged, Hungary*

Email: zcupic@ihm.bg.ac.rs

Reaction systems are wide but specific class of dynamical systems, where stoichiometric relations govern the contributions of the individual reaction rates to the overall rate of changes in state variables, which are usually concentrations of some reactive species. Hence, the stability analysis of these systems is also specific and require use of specific tools like the Stoichiometric Network Analysis (SNA).¹ In models of reaction systems, various rate laws may be used, but mass action principle is the most common. It is based on fundamental principle, that the rate of reaction is proportional to the concentrations of the reacting substances. As a result, rates of individual steps are power functions of concentrations as the state variables, and overall rates are obtained as linear combinations of such simple monomial terms. Nevertheless, final expressions are nonlinear as a rule. Moreover, number of independent equations and variables may be very large.

Numerical simulations based on efficient algorithms for integration of systems of ordinary differential equations is often the best way to analyse dynamical states of the reaction systems. However, the main model parameters – rate constants are generally unknown. Therefore, more general approach is required to evaluate possible dynamic states depending on unknown values of the rate constants. SNA is probably unique tool that may provide such general results on so complex objects as reaction systems are. Large number of reaction steps and reaction species may be limiting for application of the SNA, but several approximations are available to attain some result on instability condition even for very large systems with dozens of reactions and reaction species.

Analytical expressions for the instability condition can be easily calculated as a function of the rate constants, and then tested by comparing them with the results of the numerical simulations obtained for selected parameter values. For this purpose, bifurcation analysis and continuation of bifurcation points may lead to crucial results.

Simple model known as three variable autocatalator² will be used to illustrate the method.

Acknowledgements: Support by the Ministry of Education, Sciences and Technology of Republic of Serbia, Grants 172015 and 45001, is gratefully acknowledged.

References

- [1] Željko Čupić, Vladimir Marković, Ana Ivanović, Ljiljana Kolar-Anić, “Modeling of the Complex Nonlinear Processes: Determination of the Instability Region by the Stoichiometric Network Analysis” in: Christopher R. Brennan, Ed. *Mathematical Modelling*, Nova Science Publishers Inc., New York, (2013) pp. 111-178.
- [2] Peng B, Scott S. K, Showalter K, Period doubling and chaos in a Three-Variable Autocatalator, *Journal of Physical Chemistry* 94:5243-5246, 1990.

Cosmological Solutions of a New Nonlocal Gravity Model

Ivan Dimitrijevic

Faculty of Mathematics, University of Belgrade,

Studentski trg 16, Belgrade, Serbia

Email: ivand@matf.bg.ac.rs

Despite of numerous significant phenomenological confirmations and many nice theoretical properties, General Relativity (GR) is not final theory of gravity. Problems mainly come from quantum gravity, cosmology and astrophysics. For example, if GR is applicable to the universe as a whole and the universe is homogeneous and isotropic, then it follows that the universe contains about 68% of dark energy, 27% of dark matter and only about 5% of visible matter. However, validity of GR at the very large cosmic scale is not verified, as well as dark matter and dark energy are not yet observed in laboratory experiments. Also, GR contains cosmological singularity. These and some other problems give rise to investigate extensions of GR.

In this talk, we present modification of GR extending by nonlocal term $\sqrt{R - 2\Lambda} \mathcal{F}(\square) \sqrt{R - 2\Lambda}$, where $\mathcal{F}(\square)$ is an analytic function of the d'Alembert operator. The choice of $\mathcal{F}(\square)$ in the analytic form is motivated by existence of analytic expressions with d'Alembertian in string field theory and p-adic string theory.

We have found some exact cosmological solutions of the corresponding equations of motion without matter and with $\Lambda \neq 0$. One of these solutions is $a(t) = A t^{2/3} e^{\frac{\Lambda}{14} t^2}$, which contains properties similar to an interplay of the dark matter and the dark energy. For this solution we computed some cosmological parameters which are in good agreement with their values in the standard Λ CDM model. This nonlocal gravity model has not the Minkowski space solution. Also, some constraints on function $\mathcal{F}(\square)$ are obtained.

This is joint work with Branko Dragovich, Alexey Koshelev, Zoran Rakic and Jelena Stankovic, and based on the recent paper arXiv:1906.07560 [gr-qc].

Nonlinear Effects in RSII Cosmological Models with Tachyon Field

Goran S. Djordjević¹, Dragoljub D. Dimitrijević and Milan Milošević

Faculty of Science and SEENET-MTP Centre, University of Niš, Serbia

Email: gorandj@junis.ni.ac.rs

A model of tachyon inflation is proposed in the framework of Randal-Sundrum II model. The tachyon field that drives inflation is represented by a Dirac-Born-Infeld action on the brane. The model is based on a braneworld scenario with a D3-brane located in the ADS_5 bulk. The corresponding system of the nonlinear evolution equations is solved. The slow-roll parameters of inflation, and observational parameters: scalar spectral index (n_s) and tensor-scalar ratio (r), are computed. Obtained results are compared with observational data from the Planck collaboration. We also consider the newest results in the holographic scenario, compare and discuss analytical, numerical and observational results.

References

- [1] G.S. Djordjevic, D.D. Dimitrijevic, M. Milosevic, *On canonical transformation and tachyon-like "Particles" in inflationary cosmology*, Rom. J. Phys. 61 (2016) 99–109.
- [2] M. Milosevic, G.S. Djordjevic, *Tachyonic Inflation on (non-)Archimedean Spaces*, Facta Univ. Ser. Physics, Chem. Technol. 14 (2016) 257–274.
- [3] N. Bilić, D.D. Dimitrijevic, G.S. Djordjevic, M. Milosevic, *Tachyon inflation in an AdS braneworld with backreaction*, Int. J. Mod. Phys. A. 32 (2017) 1750039.
- [4] N. Bilić, S. Domazet, G.S. Djordjevic, *Particle creation and reheating in a braneworld inflationary scenario*, Phys. Rev. D. 96 (2017) 083518.
- [5] N. Bilić, D.D. Dimitrijević, G.S. Djordjevic, M. Milošević, M. Stojanović, *Tachyon inflation in the holographic braneworld*, J. Cosmol. Astropart. Phys. 2019 (2019) 034–034.

Nonlinear and Nonlocal Dynamics of p-Adic and Zeta Strings

Branko Dragovich,

Institute of Physics Belgrade, University of Belgrade, and Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia

Email: dragovich@ipb.ac.rs

I will present some Lagrangians for scalar p-adic and zeta strings. These Lagrangians contain nonlocality in the kinetic parts and nonlinearity in their potential parts. Nonlocality of p-adic strings is in the form of an exponential function which argument is the d'Alembert operator, while for zeta strings is presented by the Riemann zeta function. Nonlinearity of p-adic strings is polynomial and nonpolynomial of the zeta strings. Some basic properties of the relevant classical field dynamics of presented models for p-adic and zeta strings will be discussed. For details, see references [1,2,3,4].

References

- [1] B. Dragovich, Zeta-nonlocal scalar fields, *Theor. Math. Phys.* **157** (3), 1669-1675 (2008).
- [2] B. Dragovich, The p-adic sector of the adelic strings, *Theor. Math. Phys.* **163** (3), 768-773 (2010).
- [3] B. Dragovich, Nonlocal dynamics of p-adic strings, *Theor. Math. Phys.* **164** (3), 1151-1155 (2010).
- [4] B. Dragovich, Towards effective Lagrangians for adelic strings, *Fortschr. Phys.* **5-7**, 546-551 (2009).

Computer modelling of cardiovascular system

Nenad Filipovic Faculty of Engineering, University of Kragujevac, Serbia

Email: fica@kg.ac.rs

Computational study for plaque composition and initial progression using coupled continuum and discrete modelling was performed. The Navier-Stokes equations are used for blood flow in the lumen, the Darcy law is used for model blood filtration, Kedem-Katchalsky equations for the solute and flux exchanges between the lumen and the intima. The system of three additional reaction-diffusion equations is used for modeling of the inflammatory process and lesion growth model in the intima. Several examples from real patient data of the left and right coronary arteries for plaque progression are presented.

Familial cardiomyopathies (FCM) are most commonly diagnosed, or progress of the disease is monitored, through in vivo imaging, with either echocardiography or, increasingly, cardiac magnetic resonance imaging (MRI). The treatment of symptoms of FCM by established therapies could only in part improve the outcome, but novel therapies need to be developed to affect the disease process and time course more fundamentally. In SILICOFCM project we are doing in silico multiscale modeling of FCMs that would take into consideration comprehensive list of patient specific features (genetic, biological, pharmacologic, clinical, imaging and patient specific cellular aspects) capable of optimizing and testing medical treatment strategy with the purpose of maximizing positive therapeutic outcome, avoiding adverse effects, avoiding drug interactions, preventing sudden cardiac death, shortening time between the drug treatment commencement and the desired result.

We believe that with SILICOFCM project we will connect basic experimental research with clinical study and bioinformatics, data mining and image processing tools using very advanced computer models of FCM and drug and patient database and regulative in order to reduce animal and clinical studies.

Once again about nonlinear functional coordinate systems and rolling a heavy ball along their coordinate surfaces

In the memory to the scientists: my professor Danilo P. Rašković (1910-1985) and academicians: Tatomir P. Andjelić, Yu. A. Mitropolsky (January 3, 1917 - June 14, 2008), V.M. Matrosov (08.05.1932-17.04.2011) and V.V. Rumyantsev (19th July, 1921 – 10th June, 2007)

Katica R. (Stevanović) Hedrih

Department of Mechanics, Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, Serbia; and Faculty of Mechanical Engineering, University of Nis, Nis, Serbia;

E-mail: katicah@mi.sanu.ac.rs; khedrih@eunet.rs; khedrih@sbb.rs

The lecture is dedicated to scientists who have contributed to nonlinear sciences, and at the same time flushed me into refinement and research in the field of nonlinear dynamics. These are the first of my professors from whom I have taught the approaches and methods of nonlinear dynamics: Danilo P. Raskovic (August 28, 1910-1985) and academicians: Tatpmir P. Andjelic, Yu. A. Mitropolsky (January 3, 1917 - June 14, 2008). The lecture is dedicated, also, to academics V.M. Марпосов (08.05.1932-17.04.2011) andf B.B. Rumyantsev (19th July, 1921 - 10th June, 2007) who suggested and supported and recommended me to be elected to the Academy of Nonlinear Sciences in Moscow, from which the ANN Branch in Belgrade grew, of which I was a long-time scientific secretary, and from which she became the present SANN, after many transformations.

In the first part of the lecture, the results of determining the change of the base vectors of the tangent spaces of the position vectors of the kinetic point vector in moving through the three-dimensional space of the nonlinear functional coordinate system of curvilinear orthogonal coordinates will be presented. The angular velocities of rotation and the extensions of the base vectors in various orthogonal curvilinear coordinate systems are determined. Formulas of nonlinear coordinate transformations from one nonlinear functional coordinate system to other are shown (See References [1-5]). Tensor calculus applied.

The second part of the lecture presents nonlinear differential equations and their first integrals and equations of the phase trajectories of the dynamics of rolling a homogeneous ball along the coordinate surfaces of a nonlinear functional coordinate system of orthogonal curvilinear coordinates. Examples of heavy ball rolling along the coordinate surfaces of a spherical coordinate system are given (See References [6-12]).

References

1. Hedrih (Stevanović) R. K., (2018), Non-linear phenomena in vibro-impact dynamics: Central collision and energy jumps between two rolling bodies, Dedicated to memory of Professor and important scientist Ali Nayfeh (December 21, 1933-March 27, 2017). *Nonlinear Dynamics*, February 2018, Volume 91, **Issue 3**, pp 1885–1907. DOI :10.1007/s11071-017-3988-x.
2. Hedrih (Stevanović) R. K., (2010), Visibility or appearance of nonlinearity, *Tensor*, N.S. Vol. 72, No. 1 (2010), pp. 14-33, #3. Tensor Society, Chigasaki, Japan, ISSN 0040-3504.
3. Hedrih (Stevanović) R. K., (2012), Tangent space extension of the position vectors of a discrete rheonomic mechanical system, *Professor N. R. Sen Memorial Lecture*, Bulletin of the Calcutta

- Mathematical Society Volume 104, No.2(2012) pp. 81-102. Bull.Cal.Math. 104 (2) 81-102 (2012).
4. Hedrih (Stevanović) R. K., (2014), Angular velocity and intensity under change of basic vectors of position vector of tangent space of a material system kinetic point –Consideration of the difference between linear and nonlinear transformations, To memory of academician Vladimir Metodieich Matrosov (May 8, 1932-April 17, 2011) President of Academy of nonlinear Sciences. *Tensor*, Vol. 75, No. 1 pp. 71-93. Tensor Society (Tokyo), c/o Kawaguchi Inst. of Math. Soc. , Japan.
 5. *J. H. Heinbockel*, Introduction to Tensor Calculus and Continuum Mechanics, *Department of Mathematics and Statistic Old Dominion University*, Copyright c 1996 by J.H. Heinbockel. All rights reserved. Reproduction and distribution of these notes is allowable provided it is for non-profit purposes only.
 6. Hedrih (Stevanović) R. K., (2018), Non-linear phenomena in vibro-impact dynamics: Central collision and energy jumps between two rolling bodies, Dedicated to memory of Professor and important scientist Ali Nayfeh (December 21, 1933-March 27, 2017). *Nonlinear Dynamics*, February 2018, Volume 91, **Issue 3**, pp 1885–1907. DOI :10.1007/s11071-017-3988-x.
<https://link.springer.com/article/10.1007/s11071-017-3988-x>
 7. Hedrih (Stevanović) R. K., (2017), Central collision of two rolling balls: theory and examples, *Advances in Theoretical and Applied Mechanics*, Vol. 10, 2017, no. 1, 33-79.
<https://doi.org/10.12988/atam.2017.765>.
 8. Hedrih (Stevanović) R. K., (2017), Dynamics of Impacts and Collisions of the Rolling Balls, *Dynamical Systems: Theoretical and Experimental Analysis*, Springer Proceedings in Mathematics & Statistics, **Volume Number:** 182, Chapter 13, pp. 157-168. © Springer, Part of Springer Science+Business , ISBN 978-3-319-42407-1. ISSN 2194-1009 ISSN 2194-1017 (electronic)
 9. Hedrih (Stevanović) R. K., (2016), *Vibro-impact dynamics in systems with trigger of coupled three singular points: Collision of two rolling bodies*, The 24th International Congress of Theoretical and Applied Mechanics (IUTAM ICTAM 2016), Montreal, Canada, 21 - 26 August, 2016, Book of Papers, pp. 212 -213. IUTAM permanent site. ISBN: NR16-127/2016E-EPUB; Catalogue Number: 978-0-660-05459-9.
 10. Hedrih (Stevanović) R. K.,(2017), Generalized rolling pendulum along curvilinear trace: Phase portrait, singular points and total mechanical energy surface, *Computer Algebra Systems in Teaching and Research*, Edited by Alexander Prokopenya and Agnieszka Gil-Swidzka, Publisher Siedlce University of Natural Sciences and Humanities (Siedlce, Poland), 2017, Vol. VI, pp. 204-216. ISSN 2300-7397. <http://www.castr.uph.edu.pl>.
 11. Hedrih (Stevanović) R. K.,(2006/2007), Predavanja: Mehanika III - Dinamika školska godina 2006/2007 (Lectures: Mechanics III- Dynamics University year 2006/2007), Faculty of Mechanical Engineering at University of Nish, Electronic version at link: <http://www.hm.co.rs/mehanika/>;
 12. Hedrih (Stevanović) R. K., (2019), "Rolling heavy ball over the sphere in real R^3 space", *Nonlinear Dynamics*, pp 1–20 ; Springer . in press, (DOI: 10.1007/s11071-019-04947-1). *Nonlinear Dyn* (2019) 97:63–82, <https://doi.org/10.1007/s11071-019-04947-1>
 13. Hedrih (Stevanović) R. K., (2019), " Vibro-impact dynamics of two rolling heavy thin disks along rotate curvilinear line and energy analysis", *Nonlinear Dynamics*, Springer . in press, Submission NODY-D-18-02888R2; (M21a=10)\ **DOI:** **10.1007/s11071-019-04988-6**
<http://link.springer.com/article/10.1007/s11071-019-04988-6>

Gravity current behind precipitation patterns

Agota Toth¹ and Dezso Horvath²

¹*Department of Physical Chemistry and Materials Science University of Szeged*

²*Department of Applied and Environmental Chemistry University of Szeged*

Email: horvathd@chem.u-szeged.hu

By maintaining the spatial separation of reactants when running a chemical reaction, one can take advantage of the presence of various gradients. In a precipitation reaction, nucleation only occurs in narrow regions where the components mix, which are shaped by the transport processes driven by gradients of concentration, density. In flow-driven systems the spatial pattern of the precipitate is governed by the underlying flow field originating from buoyancy effects. When a denser liquid containing one component is injected into the initially stagnant solution of the other, a gravity current develops. As it advances on the bottom, it is characterized with a large convection roll at its tip, where the liquid is forced upward providing enhanced mixing.

With an instantaneous precipitation reaction, nucleation and growth are restricted to this thin zone resulting in an expanding rim.

The tip of the gravity current is elevated from the bottom because of the viscous drag exerted by the wall, creating a zone of unstable stratification. A transverse array of small convection rolls develops modulating the vertical and transverse velocity field.

With slower nucleation, solid particles sediment in areas with descending fluid leading to the final pattern characterized with evenly spaced, growing lines of precipitate. Well beyond the tip the vertical component of fluid velocity is negligible above the gravity current, leaving diffusion the only transport mode for mixing the components.

Hence on the longest time scale precipitate can also form in this homogeneous horizontal layer.

Symmetries, invariant manifolds and integrability of natural mechanical systems on Stiefel varieties

Božidar Jovanović, Mathematical Institute SANU, Belgrade, Serbia

Email: bozaj@turing.mi.sanu.ac.rs

We study natural mechanical systems on Stiefel varieties. We give geometrical description of the flows and construct integrable discrete models. The results are obtained in collaboration with Yuri Fedorov (UPC, Barcelona).

Formation of differential equations to describe the dynamics of a reaction system

Ljiljana Kolar-Anić¹ and Željko Čupić²

¹*University of Belgrade, Faculty of Physical Chemistry, Belgrade, Serbia;*

²*University of Belgrade, Institute of Chemistry, Technology and Metallurgy, Department of Catalysis and Chemical Engineering, Belgrade, Serbia*

Email: ljiljana.kolar.anic@gmail.com

If we want to analyze the dynamic states of a reaction system, we first need to identify this reaction system with the main reaction species in it together with relations between them. In other words, the minimal reaction model must be defined.

Temporal dynamics of the concentrations of all species is governed by a system of ordinary differential equations (ODEs) derived from the reaction model by the law of mass action. Law of mass action is the principle which tell us that the rate of reaction is proportional to the concentrations of the reacting substances. The rate of reaction is the rate at which reactants are converted into products.

Formation of differential equations necessary for mathematical analysis of dynamic states of a considered nonlinear reaction system can be illustrated on one example¹. (Table 1) On the left side we can see the model of mechanism of reaction system. The corresponding rate constants are indicated on the arrows. If the concentration of species X_1 , X_2 , X_3 and X_4 , are denoted by x_1 , x_2 , x_3 and x_4 , the rates of particular reactions under isothermal conditions are given in the middle and related differential equations that describe time evolution of reaction species are presented on the right side.

Table 1. Model of the mechanism of a reaction system, the rates of particular reactions and related differential equations that describe time evolution of reaction species

$\xrightarrow{k_0} X_1$	$r_0 = k_0$	
$\xrightarrow{k_m} X_3$	$r_m = k_m$	$\frac{dx_1}{dt} = k_0 - k_1 x_1$
$X_1 \xrightarrow{k_1} X_2$	$r_1 = k_1 x_1$	$\frac{dx_2}{dt} = k_1 x_1 - k_2 x_2 - k_3 x_2 - k_4 x_2 x_4^2 - k_6 x_2$
$X_2 \xrightarrow{k_2} X_4$	$r_2 = k_2 x_2$	
$X_2 \xrightarrow{k_3} X_3$	$r_3 = k_3 x_2$	$\frac{dx_3}{dt} = k_m + k_3 x_2 - k_5 x_3 x_4^2$
$X_2 + 2X_4 \xrightarrow{k_4} 3X_4$	$r_4 = k_4 x_2 x_4^2$	
$X_3 + 2X_4 \xrightarrow{k_5} X_4$	$r_5 = k_5 x_3 x_4^2$	$\frac{dx_4}{dt} = k_2 x_2 + k_4 x_2 x_4^2 - k_5 x_3 x_4^2 - k_7 x_4$
$X_2 \xrightarrow{k_6} P_1$	$r_6 = k_6 x_2$	
$X_4 \xrightarrow{k_7} P_2$	$r_7 = k_7 x_4$	

Such evaluated system of differential equation can be analyzed by standard mathematical procedure with aim to obtain time evolution of all species, dynamic states of the system and related bifurcations.

The well defined model can be used for various applications by addition of new species and new reactions. Differential equations must follow the changes in the model.

Acknowledgements:

Support by the Ministry of Education, Sciences and Technology of Republic of Serbia, Grants 172015 and 45001, is gratefully acknowledged.

References

- [1] Jelić S, Čupić Ž, Kolar-Anić Lj. Mathematical modelling of the hypothalamic-pituitary-adrenal system activity. *Mathematical Biosciences* **197**:173-187, 2005.

Nonlocal solitons

Wieslaw Krolikowski

Texas A&M University at Qatar, 23874 Doha, Qatar

Email: wieslaw.krolikowski@qatar.tamu.edu

In this talk I will discuss propagation and localization of optical beams in nonlinear media with spatially nonlocal nonlinearity. I will show that the nonlocality has a profound effect on propagation of beams leading to such effects as suppression of collapse and stabilization of fundamental as well as higher order solitons, including vortex solitons. I will also show how the competition between different types of nonlocal nonlinearity leads to anomalous interaction of spatial solitons and spontaneous pattern formation in optical beams.

Analysis of reaction fronts observed in autocatalytic systems with reversible reactions

Stevan Maćešić^{1,3}, Ágota Tóth¹, Dezső Horváth²

¹ *Department of Physical Chemistry and Materials Science, University of Szeged, Rerrich Béla tér 1, Szeged H-6720, Hungary*

² *Department of Applied and Environmental Chemistry, University of Szeged, Rerrich Béla tér 1, Szeged H-6720, Hungary*

³ *University of Belgrade, Faculty of Physical Chemistry, Studentski trg 12-16, 11000 Belgrade, Serbia*

Email: stevan.macesic@ffh.bg.ac.rs

When an autocatalytic reaction takes place in an unstirred thin layer of solution, reaction-diffusion systems (RD) are formed, and nonlinear phenomena such as propagating reaction fronts can be observed. The main characteristics of these fronts are constant propagation velocity and constant shape. RD systems and emerging reaction fronts are essential for normal functioning of many biological processes, such as regulation of reactive oxygen species (ROS) dynamics in rhizosphere. Therefore, model of ROS dynamics [1] represented by complex reaction-diffusion network was analyzed with an aim to explain the nature of reaction fronts detected while the system undergoes reduction. By carrying out numerical investigation of the considered model it was found that reaction diffusion fronts are the result of oxidation process. It was found that fronts detected while the system undergoes reduction are diffusion-driven, their constant shape is the result of autocatalysis opposing diffusion. Further analysis has shown that this is a general behavior of autocatalytic systems containing reversible reactions.

Acknowledgments

The financial support of the National Research, Development and Innovation Office (K119795) and GINOP-2.3.2-15-2016-00013 projects and Ministry of Education, Sciences and Technology of Republic of Serbia (Grant 172015)

References

1. Taran, O., Patel, V., & Lynn, D. G. (2019). Small molecule reaction networks that model the ROS dynamics of the rhizosphere. *Chemical communications*, 55 (25), 3602-3605.

The brachistochronic motion of a heavy ball rolling along an imperfect rough surface

Obradović A., Šalinić S., Mašinski fakultet Univerziteta u Beogradu

Email: aobradovic@mas.bg.ac.rs

The problem of brachistochronic motion of a heavy uniform ball rolling without slip along the upper outside surface of an imperfect rough stationary sphere, is solved. The control forces are located in the tangential plane of the other sphere, along which the center of the ball is moving, and their total power equals zero in the brachistochronic motion of the mechanical systems. Thus, during motion the total mechanical energy is maintained as the projection of the ball's angular velocity onto the radial direction. The initial value of the mechanical energy is specified, and the initial and final position of the ball's center is defined by the spherical coordinate system angles, the ball orientation (Euler angles) not being considered in this problem. Such mechanical system is nonholonomic, and dynamic equations in this paper are derived using the general theorems of mechanics.

In the first part of the paper the determination of the brachistochronic motion is solved as the problem of optimal control using Pontryagin's maximum principle. Three projections of the ball's angular velocity onto the base vectors of the spherical coordinate system are taken for controls. The two-point boundary value problem, which is reduced to the two-parameter shooting of one coordinate of the conjugate vector and end moment, is solved by the least squares solution. Based on the estimates of initial values for one multiplier and the fact that the end moment is positive, it is graphically shown that the obtained solution presents the global minimum time (the one with the minimum time of motion of all possible solutions that satisfy the Maximum principle). It is shown that there is no angular velocity projection onto the radial direction. This solution corresponds to the motion of the heavy ball along a perfect rough sphere, because it is necessary to ensure unrealistically high Coulomb coefficient of sliding friction. It represents the maximum possible ratio between the tangential and the normal components of the reaction of constraint with which the sphere is acting upon the ball. The expression is derived for the ratio between these components, depending on the initial energy and polar angle.

The second part provides the following discussion: if the constraint between the sphere and the ball is imperfectly rough, the formulation of the optimal control problem should include restrictions to the ratio between the tangential and the normal components of the reaction of constraint. Here, the problem of optimal control is formulated in such way that the tangential component of the reaction of constraint is taken for the control, with the restriction resulting from Coulomb laws of sliding friction. The problem thus formulated belongs to the theory of singular optimal controls, and the solution that satisfies the Maximum principle consists of a singular part at the beginning of motion and a non-singular part, during which the ratio between mentioned components has maximum possible value that concrete surfaces can achieve. It is shown that mutual detachment of the bodies cannot occur before their mutual slipping at the contact point. A corresponding numerical example is given, with graphical representation of the effects of initial energy values, Coulomb coefficient and ultimate height of the ball center on the solution structure, in this case. The review highlights regions where it is possible to obtain a singular control across the entire motion or a combination of a singular and non-singular part of the optimal trajectory.

Nonlinear Model of Fluid Diffusion in Porous Solids

Nickolay Ostrovskii, Euro Gas, doo, Senčanski put 75, 24000 Subotica,
Email: n-ostr@yandex.ru

In many processes such as adsorption, drying, solid fuel combustion, and catalytic reactions, the mass transfer of fluid in porous media is often the rate limiting step. Despite of heterogeneous porous structure, and different mechanisms of transport, the mass transfer is often characterized by only one effective diffusion coefficient, which is a lumped parameter.

In some models (especially in drying and adsorption) such effective diffusivity is represented as a function of moisture content, which is contradictory to Fick's diffusion equation.

Another approach is based on assumption of heterogeneous porous structure and on models of multi-phase mass transfer. Such models are applied for drying of fruits because are based on their cellular structure.

Recently, a generalized approach has been developed to multicomponent diffusion based on the idea of the reaction mechanism borrowed from chemical kinetics [1]. Authors have proposed also the suitable equations for Fick's diffusion and for Cell-jump formalism, as well as complex balance conditions for Mass Action Law application to diffusion.

Based on these approaches, we have proposed the nonlinear model of moisture diffusion in a poly-disperse porous particles [2]. The model of drying has been derived assuming three types of moisture inside porous particles, which are corresponded to three sizes of pores.

The diffusion flux is expressed in Onsager form $J_i = -\sum_j L_{ij} X_{ji}$, according to concept of linear non-equilibrium thermodynamics, but with non-linear thermodynamic force X_{ji} . The driving force is considered as a product $X_{ji} = x_j (1-x_i)$ of fullness degree by moisture of one type of pores x_j and emptiness of another type $(1-x_i)$.

The model is applied to drying of polymer porous particles filled with solvent. The model in a form of differential equations, and results of experiments are presented in this report.

References

1. A.N. Gorban, H.P. Sargsyan, H.A. Wahab, – Quasichemical Models of Multicomponent Nonlinear Diffusion – Math. Model. Nat. Phenom., 2011, **6**, 184-262
2. N.M. Ostrovskii, K. Mészáros-Szécsényi– Application of Quasi-Chemical Reaction Approach to Kinetics of Diffusion in Porous Material. – Int. Conf. Physical Chemistry, Belgrade, 2016.

Controlling Pattern Formation of Cadmium-Hydroxide System

Papp Paszkál, University of Szeged

Email: pacek97@hotmail.com

Controlling Pattern Formation of Cadmium-Hydroxide System Paszkál Papp, Ágota Tóth, Dezső Horváth Department of Physical Chemistry and Materials Science, University of Szeged Chemical reactions occurring under unstirred conditions can exhibit various types of exotic behaviour. One intriguing example is the Liesegang phenomenon where precipitation reaction is coupled by diffusion resulting in precipitation rings in a two dimensional arrangement. During the reaction, precipitate bands and depletion zones form behind the propagating reaction front. Experimentally has been observed that pattern formation in the cadmium-hydroxide system can be controlled by altering the front propagation velocity with the immobilization of hydroxide ions. The aim of this study is to determine the governing principles of the pattern formation by numerical analysis. For the modelling we use the spinodal phase separation theory with solving the Cahn-Hilliard equation. We not only determine the major driving force but also characterize the evolving structures.

Nonlinear Dynamics, Chaos and Complex Systems: A historical perspective

Miguel A. F. Sanjuán

Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física

Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain

Email: miguel.sanjuan@urjc.es

When we talk about dynamics, we do not only understand the motion of celestial bodies and solid mechanical systems, but any changes with respect to time of one or more variables. From that point of view, we can find dynamics everywhere, in any field of science. Thus, now we have a more general vision, including stock market movements and economic variables, concentration changes in chemical reactions, changes in physiological, biological and medical variables, action potentials of neurons, etc ... providing a more interdisciplinary perspective.

The various interactions between the constituent parts of a physical system and their feedback mechanisms, are a source of nonlinearity and complexity, which added to the sensitivity dependence to initial conditions which is a hallmark of chaotic behavior, constitutes a change of perspective in dynamical systems with important consequences for the understanding of science.

I will give a historical perspective of Nonlinear Dynamics, Chaos Theory and Complex Systems, including some of the different sources that have contributed to the construction of the discipline as we know it today. Among them, the three-body problem in celestial mechanics, turbulence in fluid dynamics, irreversibility and fundamentals of statistical physics and the logistic map and population dynamics in biology. Many schools of mathematics and physics have played an essential role in the historical development of the subject, including the French, Russian, Japanese and American school. The knowledge of this historical perspective allows us to understand the breadth of the discipline itself and the multiple interdisciplinary applications to various fields of science.

T-duality in superstring theory

Branislav Sazdovic, Institute of Physics, University of Belgrade

Email: sazdovic@ipb.ac.rs

T-duality plays an important role in understanding the M-theory. That is why we will consider several important steps in better understanding T-duality. First we are going to consider Buscher approach to T-duality and its generalization to the case of coordinate dependent background.

Then will conclude that generalized Buscher approach leads to non-geometry. Finally, using double space we will be able to unify all T-dual theories but only in case of bosonic string.

Nonlinear self-organization of solitons and optical soliton-tweezers in nanocomposites and suspensions of nanoparticles and nanographenes

*Vladimir Skarka, University of Angers, France, and Institute of Physics of Belgrade
Email: vskarka@yahoo.com*

Nonlinear self-organization of solitons and optical soliton-tweezers in nanocomposites and suspensions of nanoparticles and nanographenes Vladimir Skarka Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia Laboratoire de Photonique d'Angers, University of Angers, 49045 Angers, France Laser beams and pulses are powerful tools for tweezing, photobiomodulation, and manipulation of soft matter including colloidal nanosuspensions, emulsions, foams, as well as all kinds of biomedica like myosin, kinesin, ribosomes, liposomes, bacteria, viruses, blood, and a variety of living cells in body water [1]. The laser modifies the nonlinear mater passing through. Simultaneously, the modified mater acts to the light altering it by a feedback mechanism. Therefore, light is controlled by light through interaction with nonlinear mater. Laser stability and precision, using this feedback mechanism, are of crucial importance not only for brain surgery but also for nondestructive diagnostics. In order to achieve the necessary dynamical stability, the promising mechanism is the self-structuring of the light into localized solitons via nonlinear interaction inside the colloidal nanosuspensions and other varieties of soft matter [2-7]. Tweezing solitons stable propagation is self-organized by the balance of antagonist effects, i.e., beam self-focusing and self-defocusing [2-6]. The high frequency pressure force of the laser beam either attracts or repels the nanoparticles from the field region, depending if their optical index of refraction is larger or smaller than the background one [3]. In both cases, the nanoparticles density modification results in the nonlinear increase of effective index of refraction inside the beam making it self-focusing. We use this self-focusing effect to establish theoretically, numerically and experimentally the self-organization of soliton-tweezers as a novel kind of dynamically reconfigurable self-collimated tweezing facilities. Nanocomposites and colloidal nanosuspensions involving graphene and various two-dimensional materials can be tweezed using spatiotemporal dissipative solitons and multidimensional vortex solitons [4, 5]. Soliton-tweezer propagating through a cuvette filled by nanographene suspended in pure water, is described by a novel complex intensity equation based on the synergy between theory and experiment via numerical simulations. Established soliton-tweezers are able to trap and photobiomodulate micro and nanoparticles in body water and other soft mater in a noninvasive way of interest for medical and biological applications.

References

- [1] A. Ashkin, J. M. Dziedzic, T. Yamane, *Nature* 330, 769 (1987).
- [2] V. Skarka, N. B. Aleksić, *Phys. Rev. Lett.* 96, 013903 (2006).
- [3] V. Skarka, N. B. Aleksić, H. Leblond, B. A. Malomed, and D. Mihalache, *Phys. Rev. Lett.* 105, 213901 (2010).
- [4] V. Skarka, N. B. Aleksić, M. Lekić, B. N. Aleksić, B. A. Malomed, D. Mihalache, and H. Leblond, *Phys. Rev. A* 90 (2), 023845 (2014).
- [5] V. Skarka, N. B. Aleksić, W. Krolikowski, D. N. Christodoulides, S. Rakotoarimalala, B. N. Aleksić, M. Belić, *Opt. Express* 25, 284183 (2017).
- [6] E. L. Falcao-Filho, C. B. de Araújo, G. Boudebs, H. Leblond, and V. Skarka, *Phys. Rev. Lett.* 110, 013901 (2013).
- [7] V. Skarka, M. Lekić, A. Kovačević, B. Zarkov, N. Romčević, *Opt. Quant. Electron.* 50, 37 (2018).

Linear and Nonlinear Effects in Tachyon Inflation in the Holographic Braneworld

Marko Stojanović

Faculty of Medicine, University of Niš, Serbia

Faculty of Sciences and Mathematics, University of Niš, Serbia

Email: marko.stojanovic@pmf.edu.rs

We consider a model of inflation based on a holographic braneworld scenario with a D3-brane located at the holographic boundary of an asymptotic Anti de Sitter bulk. Inflation is driven by a tachyon field described by the Lagrangian of the Dirac-Born-Infeld type. We solve the nonlinear equations of motion analytically in the slow-roll regime for the tachyon exponentially attenuating potential, and we find numerical solution of the exact equations. The difference between numerical and analytical results and dependence of initial conditions is analyzed. We also calculate some of the observational parameters of inflation and compare the results with Planck 2018 data.

References

- [1] M. Milošević, D.D. Dimitrijević, G.S. Djordjević and M.D. Stojanović, Dynamics of Tachyon Fields and Inflation – Comparison of Analytical and Numerical Results with Observation, *Serbian Astronomical Journal* (2016), Vol. 192, 1-8.
- [2] D.D. Dimitrijevic, N. Bilić, G.S. Djordjevic, M. Milosevic, M. Stojanovic, Tachyon scalar field in a braneworld cosmology, *Int. J. Mod. Phys. A* (2018), Vol 33, No. 34, 1845017.
- [3] N. Bilić, D.D. Dimitrijević, G.S. Djordjevic, M. Milošević, M. Stojanović, Tachyon inflation in the holographic braneworld, *J. Cosmol. Astropart. Phys.* 2019 (2019) 034–034.
- [4] M. Milošević, N. Bilić, D.D. Dimitrijević, G.S. Djordjević and M. Stojanović, Numerical Calculation of Hubble Hierarchy Parameters and Observational Parameters of Inflation, *AIP Conference Proceedings* (2019), 2075, 090009.
- [5] M. Stojanović, Neven Bilić, D.D. Dimitrijević, G. S. Djordjevic, M. Milosević, Inflationary RSII model with matter in the bulk and exponential potential of tachyon field, *Facta Universitatis: Physics, Chemistry and Technology* (2019), Vol 17.

Bond Portfolio Management in Insurance Industry – Implications of Solvency II Regulation

Branko Urošević, Faculty of Economics, University of Belgrade

Email: imqfbrankourosevic@gmail.com

We develop a novel approach to the bond portfolio optimization problem faced by insurance companies that are subject to the Solvency Capital Requirement (SCR) regulation. The SCR regulation is a combination of restrictions based on the characteristics of bonds, as well as a stress test on the benchmark yield curve with associated capital requirements. The optimization problem of determining the portfolio weights for each bond is non-differentiable and non-convex due to these restrictions. We tackle the complexity of the optimization problem using the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to obtain the yield to maturity (YTM)-SCR efficient portfolios. The characteristics of the estimated efficient portfolios are examined in different scenarios for the stressed benchmark yield curve. Our findings suggest a very high concentration in all of the estimated YTM-SCR efficient portfolios. The efficient portfolios are dominated by short term and BBB rated bonds. The lack of diversification and over-exposure to bonds with higher credit risk in different market scenarios are weaknesses of the Solvency II regulation with possible unintended consequences for both the insurance industry and bond markets.

Founder of Modern Theoretical and Mathematical Physics

(to the 110th anniversary of M.M. Bogolyubov birthday)

A.G. Zagorodny

*Bogolyubov Institute for Theoretical Physics, 14-b Metrolohichna Str, 03680, Kiev,
Ukraine*

Email: azagorodny@bitp.kiev.ua

"Science is the main and only goal in my life."

Mykola Bogolyubov

The 110th anniversary of Mykola Bogolyubov provides a good opportunity to recollect the scientific achievements of this brilliant scientist and to discuss once again his invaluable contribution to the development of various fields of theoretical and mathematical physics – nonlinear mechanics, nuclear physics, quantum field theory, high energy physics, condensed matter physics, etc.

In the communication the main periods of life and scientific activity of Academician Bogolyubov are presented. His role in the progress of theoretical and mathematical physics, the formation of scientific schools and the foundation of new departments, scientific laboratories and institutes are discussed.

Bogolyubov-Born-Green-Kirkwood-Yvon-Hierarchy and Effective Grain Potentials for Dusty Plasmas

A.G. Zagorodny

Bogolyubov Institute for Theoretical Physics, 14-b Metrolohichna Str, 03680, Kiev, Ukraine

Email: azagorodny@bitp.kiev.ua

The modern state of the consistent kinetic theory of dusty plasmas (plasmas consisting of electrons, ions and charged grains continuously absorbing electrons and ions from plasma background) is discussed. The derivation of equations for microscopic phase densities of plasma particles and grains is presented. Such equations are suitable for extending the Bogolyubov-Born-Green-Kirkwood-Yvon-hierarchy to the case of dusty plasmas and to deriving the kinetic equations with regard for both elastic and inelastic particle collisions. Kinetic description of effective grain-grain potentials is also presented. The proposed theory is used to find explicit relations for the potential and electron and ion distributions near the grain with regard to their collisions and the presence of an external fields. In particular, the theory under consideration confirms the existence of the long-range asymptotics of the effective nonlinear potentials calculated numerically. Numerical solution of the nonlinear problem of dynamical grain screening is obtained. It is shown that plasma particle absorption by grain can lead to the effect of negative friction. The physical mechanisms of this phenomenon are discussed.

Transverse Transport of Magnetized Particles in Random Electric Fields

V.I. Zasenko, A.G. Zagorodny, O.M. Cherniak,

Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine

email: zasenko@bitp.kiev.ua

The study of anomalous transport of particles across ubiquitous magnetic field is an important task of plasma physics. One of the causes of anomalous transport is commonly assumed to be an electrostatic turbulence. In the limit of small correlation times of random fields transport is diffusive, and for its description a quasilinear approximation is used. In the opposite limit of large correlation times transport occurs as advection. The key problem which is strongly nonlinear is to find a transition from the Eulerian correlation function of particle velocity to the Lagrangian one that corresponds to some particular closure of equations for particle spreading. We proposed the new closure, and subsequently derived the statistical equations which describe transport of magnetized particles undergoing random isotropic electric fields in a wide range of correlation times (including two limiting cases), without use of free parameters. Direct numerical simulations were carried out to verify the transport equations. It was shown that statistical characteristics of particle ensemble obtained from simulations are consistent with solutions of transport equations for fields such as frozen one, piecewise frozen field with phase jumps, and set of random waves.

Acknowledgement

This work was supported by the National Academy of Sciences of Ukraine under the Project PK 0118U003535.

A review of recent studies on nonlinear dynamics of microtubules and DNA

Слободан Здравковић, ИИИ Винча

Email: szdjidji@vin.bg.ac.rs

Microtubules are the major part of cytoskeleton. They are involved in nucleic and cell division and organization of intracellular structure. They also serve as a network for motor proteins. There are a few models describing their complex dynamics. The recent one, extended general model, is explained [1]. Depending on used mathematical approximation we conclude that both a breather and kink-type soliton could move along the microtubule. A helicoidal Peyrard-Bishop model is one of known models describing nonlinear DNA dynamics. Recently, a continuum approximation was applied within this model. It was shown that a kink solitons could move along the chain if viscosity is taken into consideration and bell-type solitons otherwise. One of the most intriguing aspects of DNA functioning is DNA-RNA transcription. A recent work has offered an idea according which a demodulated standing soliton mode is relevant for the DNA segments where this transcription occurs [3]. This means that the moving breather becomes a demodulated one at these segments as well as a standing wave.

References

- [1] Slobodan Zdravković, Slobodan Zeković and Aleksandr N. Bugay, Extended general model of microtubules and semi-discrete approximation, submitted to Chaos.
- [2] S. Zdravković, D. Chevizovich, A.N. Bugay, and A. Maluckov, Stationary solitary and kink solutions in the helicoidal Peyrard-Bishop model of DNA molecule, Chaos 29 (2019) 053118.
- [3] S. Zdravković, M.V. Satarić, A.Yu. Parkhomenko, and A.N. Bugay, Demodulated standing solitary wave and DNA-RNA transcription, Chaos 28 (2018) 113103.